Seventh Grade Students' Meanings of Division with Whole Numbers, Fractions, & Unknowns

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IDR²eAM Project: Investigating Differentiated Instruction and Relationships between Rational Number Knowledge and Algebraic Reasoning in Middle School

- **Study:**
  - how to differentiate instruction for cognitively diverse middle school students
  - how students' rational number knowledge and algebraic reasoning are related
- **Phase I (Yrs 1 & 2):** Conducted three 18-episode after school design experiments with 6-9 cognitively diverse middle school students [ages 12-14]

Purpose of talk

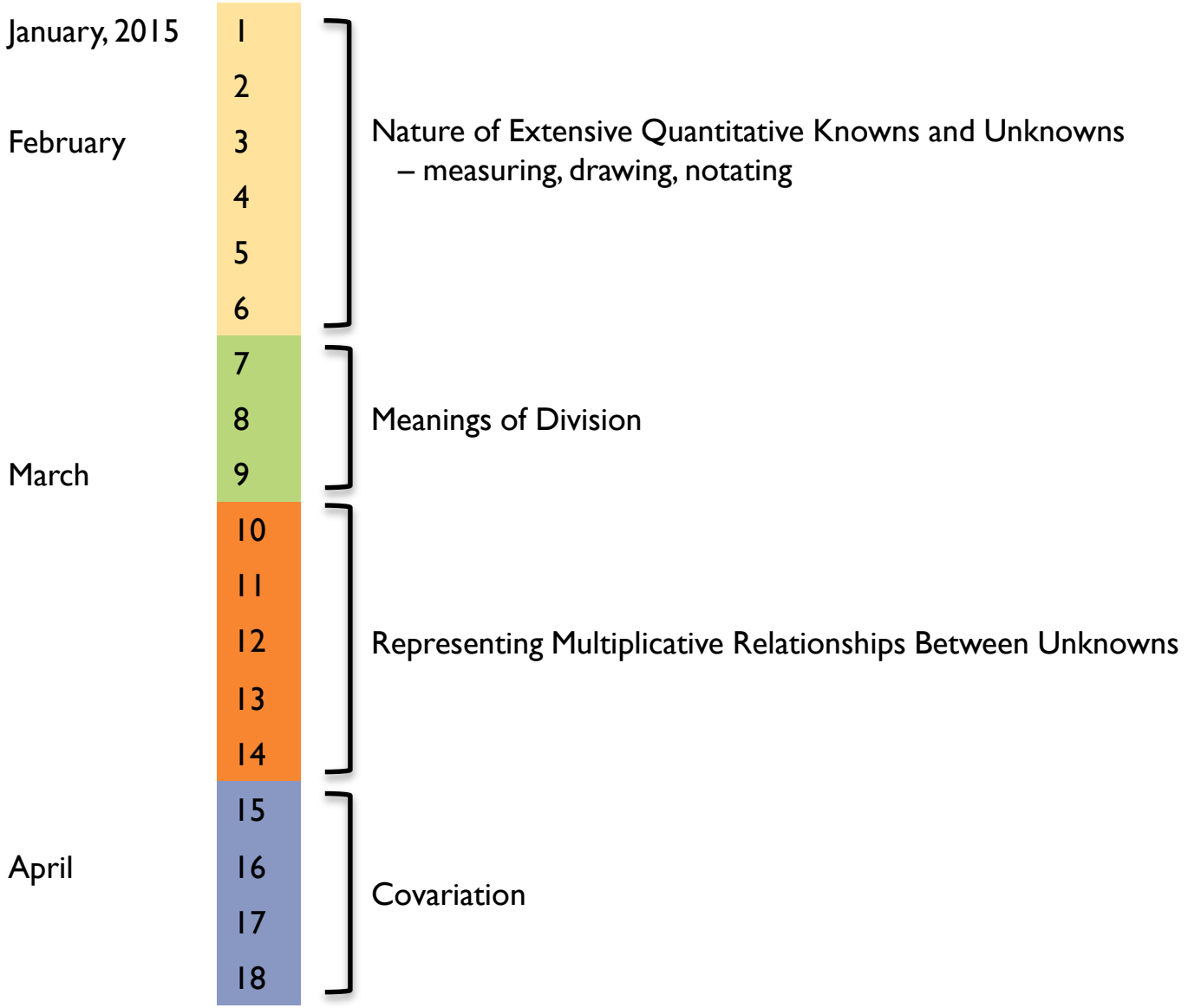
- To communicate about students' meanings of division with fractions and unknowns in the 3rd of three design experiments

- How did students think about division in representing multiplicative relationships between unknowns?

Algebraic Reasoning from a Quantitative Perspective

- Unknowns are potential measurements of quantities.

Timeline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Nature of Extensive Quantitative Knowns and Unknowns – measuring, drawing, notating
Meanings of Division
Representing Multiplicative Relationships Between Unknowns
Covariation

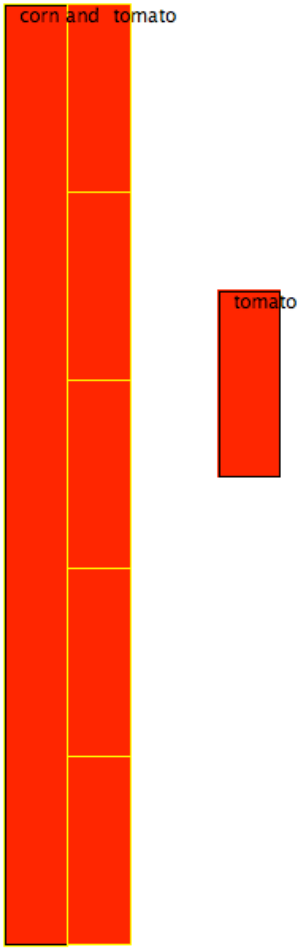
Whole number relationships between unknowns

- **Corn Stalk Tomato Plant Heights Problem.** A tomato plant and corn stalk are growing in the garden, each of unknown height.

  The height of the corn stalk measured in inches is 5 times the height of the tomato plant measured in inches.

- q = height of corn stalk
- c = height of tomato plant
- 5c = q

  q ÷ c = 5
- q ÷ 5 = c

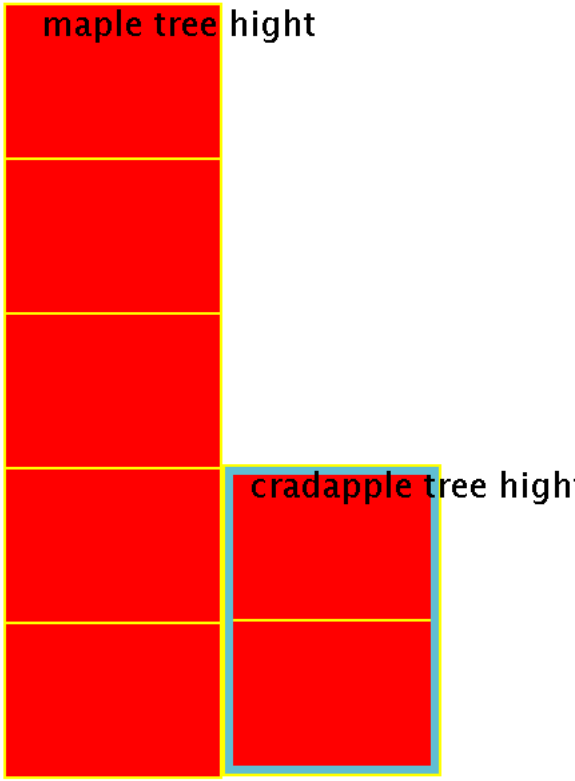


Fractional relationships between unknowns

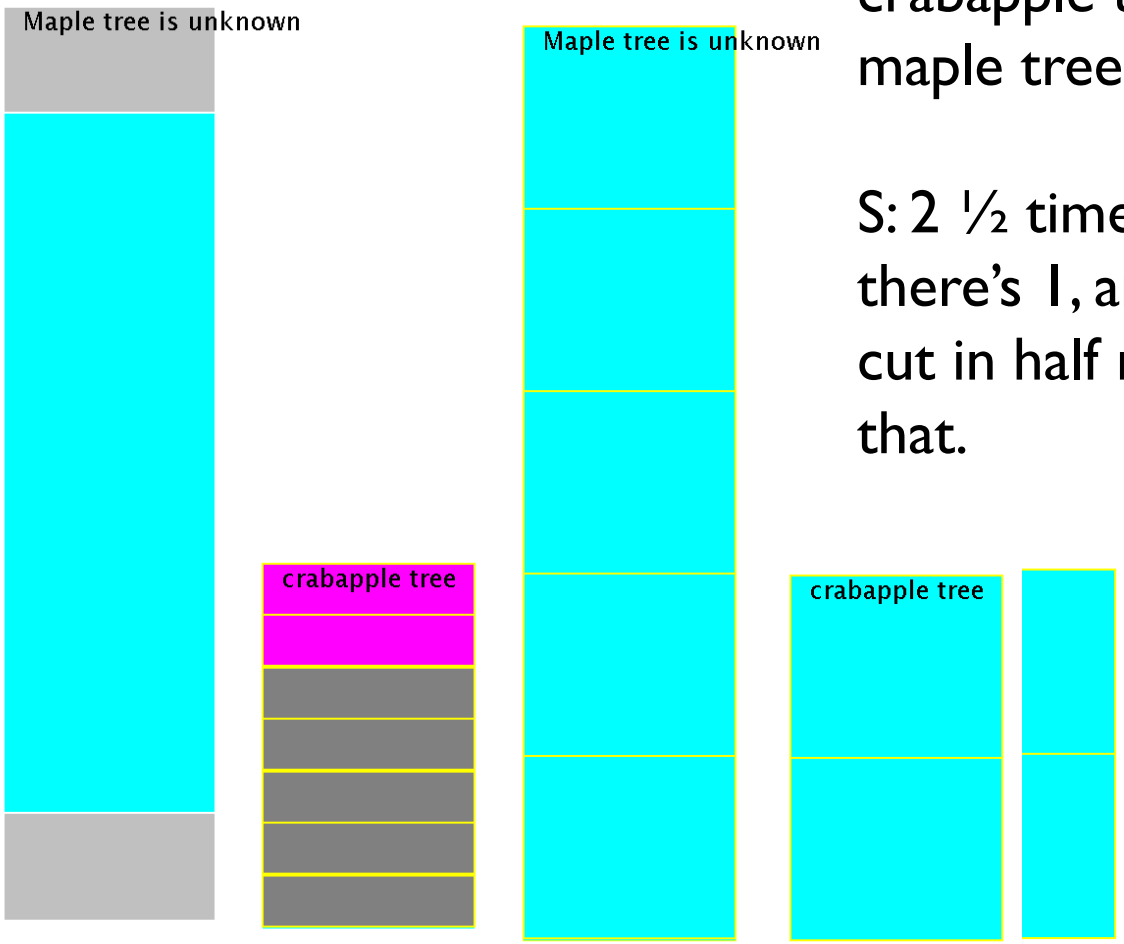
- **Tree Heights Problem.** A crabapple tree and a maple tree are growing next to West Middle School. Each tree's height is unknown.

  The principal knows that the crabapple tree's height measured in feet is 2/5 of the maple tree's height measured in feet.

- m = maple tree height
- c = crab. tree height
- m ÷ c = ?
- m ÷ c = 2½
- m ÷ c = 5/2



Heather & Symone

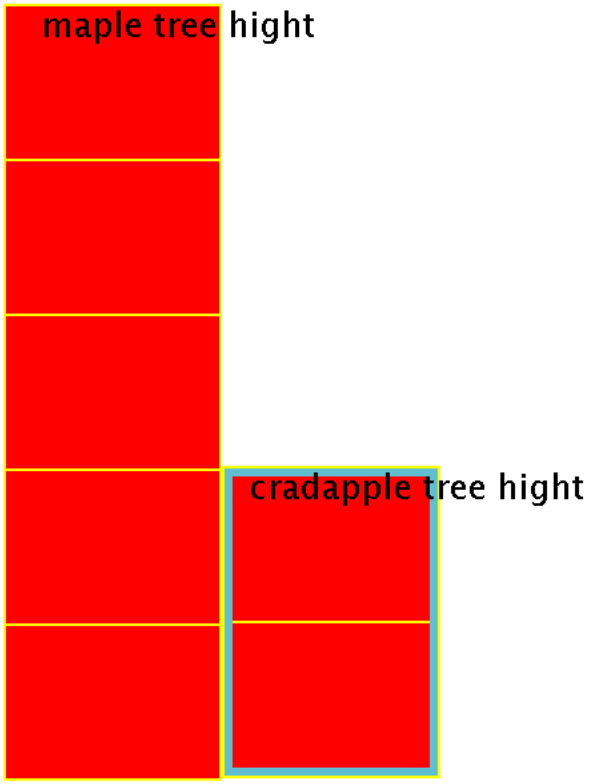


AH: Do you have a sense of how many times this crabapple tree fits into that maple tree height?

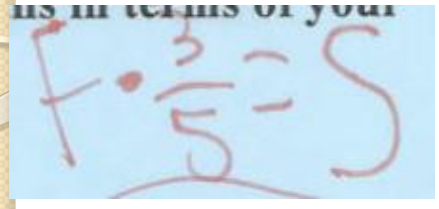
S: 2 ½ times. Because there's 1, and then this is cut in half right here like that.

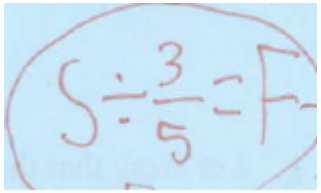
Milo

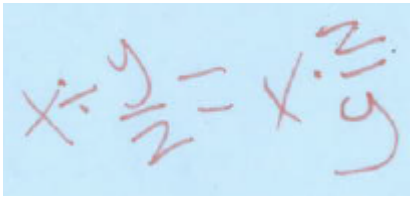
- *Initial equations:*
  - m • 2/5 = c
  - c • 0.4 = m
- *Tchr:* Recall C ÷ T = 5 discussion?
- *Milo:* Ah!
- Identified 2.5 with questioning
- *New equation:* c • 2.5 = m
- *And then after 5 min of computing with c = 4, m = 10:* m ÷ 2.5 = c

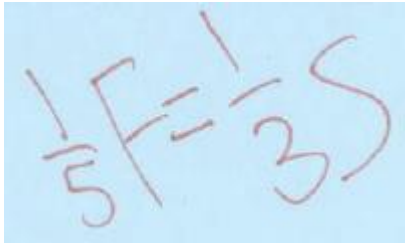


Emmett & Yujeong – a sunflower's height is 3/5 of a fern's height









- E: There's 5 unknown units [fern]. Now this is the 3, the base, which is equal to the sunflower, which is 3. So that's 3, 4, 5. That's 5 and this is the base of 3. So that's 5/3rds.
- Y: One unknown unit is 1/3 of a sunflower.
- E: And 1/5 of the fern.
- E: That's perfect. It's the best math equation ever.

Students' Meanings/Uses of Division

- Heather & Symone:
  - M ÷ C (H)
  - Measuring one quantity with the other (S)
- Milo
  - Measuring one quantity with the other made sense with prompting, but he did not come up with M ÷ C = 2 ½
  - No multiplicative inverses (no reciprocal reasoning)
- Emmett & Yujeong
  - *From:* Multiply by reciprocal "because I learned that"
  - *To:* Multiplicative inverses justified based on relationships with quantities

Danke!

- With BIG thanks to others on the **IDR²eAM** project team: Fetiye Aydeniz, Mark Creager, Ayfer Eker, Serife Sevis
- *What IDR²eAM stands for:* **I**nvestigating **D**ifferentiated Instruction and **R**elationships between **R**ational Number Knowl**e**dge and **A**lgebraic Reasoning in **M**iddle School
- **http://www.indiana.edu/~idream/**