From the IDR²eAM Project: Investigating Differentiated Instruction and Relationships between Rational Number Knowledge and Algebraic Reasoning in Middle School

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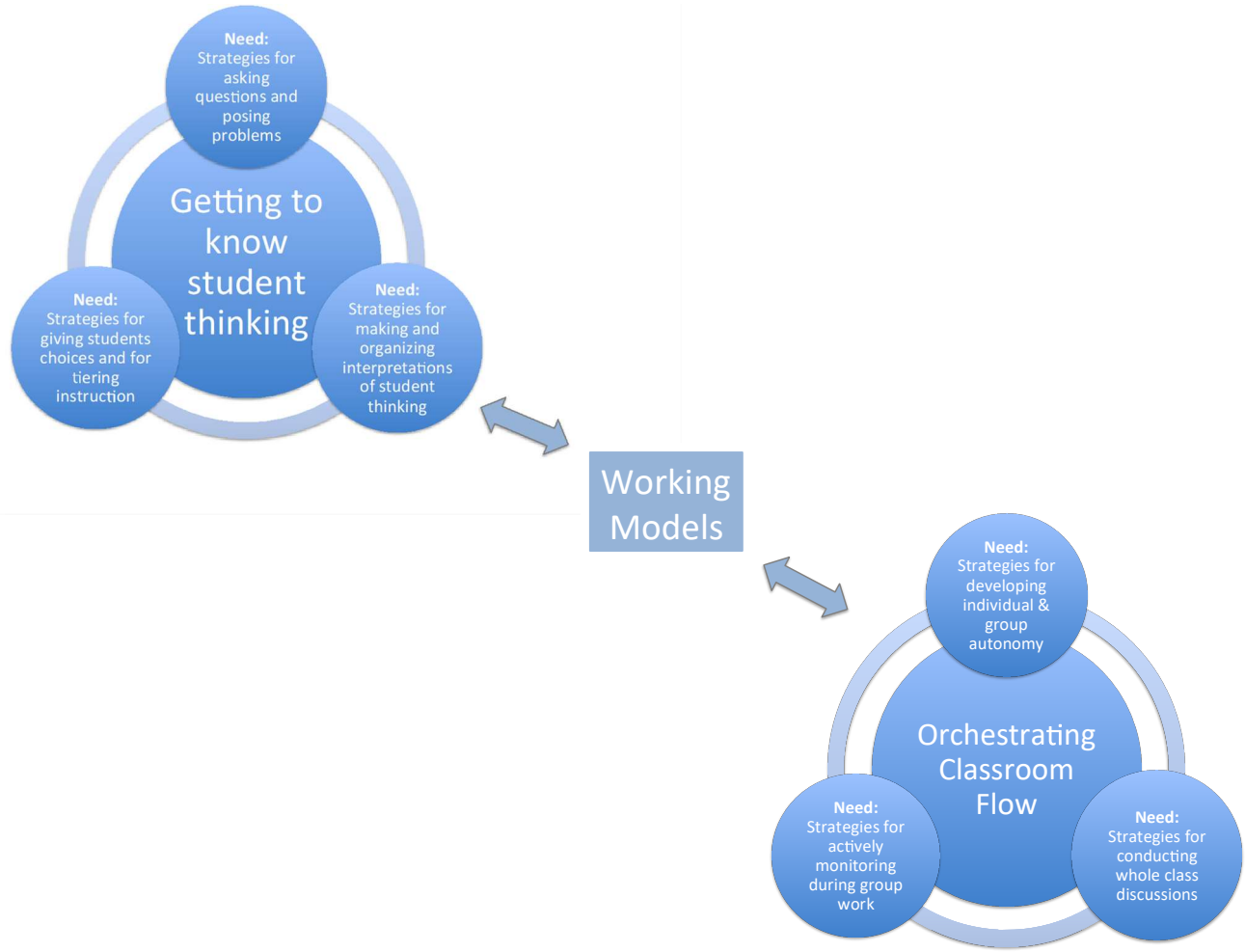
Crate Problem: There are 4 cans of juice in a package and 8 packages in a box. A crate contains 6 boxes. How many cans of juice are in a crate, and can you draw a picture to show how you know?

Partially Filled: A worker is packing one of those crates. They have already packed 2 boxes and 3 packages. How many more cans are needed to fill the crate? How will those cans be organized in the crate?



Differentiated Versions of Investigation 4.1 from Connected Math[1]

All students did the following problem:

1) Pauline the pool worker comes in to fill a pool at noon on Monday. Her hose fills the pool at 12 gallons per minute.
   a. What equation could you write to represent the amount of water in the pool, w, after t minutes since noon?
   b. How many gallons will be in the pool after 30 minutes? An hour? 90 minutes?
   c. If the pool holds 5400 gallons, how much time does it take to fill the pool?
   d. Describe the pattern of change in the amount of water in the pool and the amount of minutes that have passed.
   e. What does the graph for this situation look like? Can you determine what the graph will look like without plotting points?

Then each was assigned one of the following:

Version A	Version B
Remember Pauline the pool worker? She is still working at her job maintaining pools. 2) When Pauline comes in at noon on Tuesday to fill another identical pool, fortunately she finds that there are already 1080 gallons of water in the pool. She will still fill the pool at a rate of 12 gallons per minute. This equation expresses the amount of water, w, in the pool after t minutes since noon: $w = 12t + 1080$ a. What does the 12 mean in the equation? b. Why is 1080 added to $12t$ in the equation? c. How many gallons will be in the pool after 30 min? After an hour? d. If the pool holds 5400 gallons, how much time does it take to fill the pool? e. Describe the pattern of change in the amount of water in pool and the amount of minutes that have passed. f. What does the graph for this situation look like? Can you determine what the graph will look like without plotting points? g. An engineer comes in partway through the day and figures out how many gallons are in the pool. She does it again 15 minutes later. How much will the amount of water change in that 15 minutes? Does it matter when during the day she arrives?	Remember Pauline the pool worker? She is still working at her job maintaining pools. 3) When Pauline comes in at noon on Wednesday to fill another identical pool, the pool is empty. She gets a phone call and does not start filling the pool for 5 minutes. This equation expresses the amount of water, w, in the pool after t minutes since noon: $w = 12(t - 5)$ a. What does the 12 mean in the equation? b. Why is 5 subtracted from t in this equation? c. Write an expression for w that is equivalent to the original expression in the equation above. What information does this new expression tell you about the situation? d. How many gallons will be in the pool after 30 min since noon? After an hour since noon? e. If the pool holds 5400 gallons, how much time does it take to fill the pool? f. Describe the pattern of change in the amount of water in pool and the amount of minutes that have passed. g. What does the graph for this situation look like? Can you determine what the graph will look like without plotting points? h. An engineer comes in partway through the day and figures out how many gallons are in the pool. She does it again 15 minutes later. How much will the amount of water change in that 15 minutes? Does it matter when during the day she arrives? i. What if the engineer measures the amount of water in the pool, and then measures it again ½ of a minute later. How much will the amount of water change in that ½ minute? Explain.

4) When Pauline comes in at noon, a pool identical to the one on the other days is full. She is supposed to empty the pool, and it empties at 12 gallons per minute. This equation expresses the amount of water, *w*, in the pool after *t* minutes since noon:

$$-12t = w$$

a. Explain why the equation involves -12. What does that mean?
b. When t is 30, w is -12(30)=-360. What do the values of t = 30, w = -360 mean in the situation?
c. The pool holds 5400 gallons. Find out how much water is left in the pool 2 hours after noon.
d. Describe the pattern of change in the amount of water in the pool and the amount of minutes that have passed.
e. What does the graph for this situation look like? Can you determine what the graph will look like without plotting points?
f. An engineer measures the amount of water in the pool, and then measures it again ½ minute later. How much will the amount of water change in that ½ minute? Does it matter which ½ minute she measures during the day?

5) When Pauline comes in at noon on Friday, a pool identical to the one on the other days is full. She is supposed to empty the pool, and it empties at 12 gallons per minute. However, first she has to change the filter for the pool, which takes 10 minutes. This equation expresses the amount of water, *w*, in the pool after *t* minutes since noon:

$$w = -12(t - 10)$$

a. Explain why the equation involves -12. What does that mean?
b. Why is 10 subtracted from t in this equation?
c. Write an expression for w that is equivalent to the original expression in the equation above. What information does this new expression tell you about the situation?
d. When t is 30, w is -12(30-10)=240. What do the values t=30, w=240 mean in the situation?
e. The pool holds 5400 gallons. Find out how much water is left in the pool 2 hours after noon.
f. Describe the pattern of change in the amount of water in the pool and the amount of minutes that have passed.
g. What does the graph for this situation look like? Can you determine what the graph will look like without plotting points?
h. What is the y-intercept for this graph? What could the y-intercept mean in this situation?

[1]Lappan, G., Phillips, E. D., Fey, J. T., Friel, S. N., Grant, Y., & Stewart, J. (2014). *Connected mathematics 3*. Boston, MA: Pearson.