Tiers, not Tears! One Strategy for Differentiating in Middle School

From the IDR²eAM Project: Investigating Differentiated Instruction and Relationships between Rational Number Knowledge and Algebraic Reasoning in Middle School

Indiana University, Bloomington http://www.indiana.edu/~idream/

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7TH GRADE MATHEMATICS CLASS

SAME SPEED TASK: The blue car travels _____ miles in _____ minutes. Make the red car travel at the <u>same</u> speed as the blue car, but the red car will travel a different amount of miles and a different amount of minutes. [Later we asked them to justify their results with a picture and explanation.]

Fractions Knowledge	Orangeyness Task	Numbers
• Fractions are parts within	 Not fluidly iterating two 	• 18 miles in 3 min
wholes or parts out of	quantities as a composed unit.	• Whole number unit ratio
wholes-no length meaning		(6 miles in 1 minute)
• Beginning to think of fractions	 Iterating two quantities as a 	• 15 miles in 6 min
as lengths	composed unit.	• Mixed number unit ratio with
• Improper fractions are not		$\frac{1}{2}$ (2.5 miles in 1 minute)
numbers		
• Think of fractions as lengths	 Iterating two quantities as a 	• 15 miles in 9 min
 Improper fractions are 	composed unit	• Unit ratio hard to work with as
numbers	 Making unit ratios 	a decimal (5/3 miles in 1
		minute)

Lisa and Sara's group (transcript), Day 11: Their blue car travels 15 miles in 6 minutes.

Sara: It's impossible.

Lisa: Ms. Hackenberg, It's impossible. We give up.

AH: I'm coming back, just a second. [She talks with another group.]

Lisa [when Ms. H returns]: It's impossible, even when we do that, 15.1 and 6.1, because it's not 15 or 6, so.

AH: Okay, so you really think it's impossible?

Lisa: Yeah.

Sara: Yeah, unless you do, like...

AH: So, two cars can't go the same exact speed but go different distances and times?

Sara: They probably could, but I can't figure it out.

Lisa: When you say we can't use 15 or 6, it's kind of hard.

AH: Right. [pause] All right, well that's good to know that it's hard.

Sara: Unless you double it, and it's going the same speed, you just doubled it.

AH: Wait, what do you mean, Sara?

Lisa: 30 and 12?

AH: Try that. What does that give you?

Sara: 30 and 12?

Lisa: Yeah, it's doubling.

- AH: Double the distance and double the time. Do you think you'll go the same speed or no?
- S: I don't know. [She watches as Lisa tries these numbers in the app.] I mean they're not going the same exact speed but they're going the same speed, just...

AH: Do you think it'll be the same speed, not the same exact--?Sara [standing up]: I figured the system out!Lisa: Oh wow, it worked! Okay.AH: [laughing]: Write that down.Sara: I'm smart. What do I get?Lisa: So, do we just put tie [do they write "tie" in their tables]?Sara: Oh I did it!

8TH GRADE PRE-ALGEBRA CLASS

Differentiated Versions of Investigation 4.1 from Connected Math¹

All students did the following problem:

- 1) Pauline the pool worker comes in to fill a pool at noon on Monday. Her hose fills the pool at 12 gallons per minute.
 - a. What equation could you write to represent the amount of water in the pool, w, after t minutes since noon?
 - b. How many gallons will be in the pool after 30 minutes? An hour? 90 minutes?
 - c. If the pool holds 5400 gallons, how much time does it take to fill the pool?
 - d. Describe the pattern of change in the amount of water in the pool and the amount of minutes that have passed.
 - e. What does the graph for this situation look like? Can you determine what the graph will look like without plotting points?

Then each was assigned one of the following:

Version A	Version B		
Remember Pauline the pool worker? She is	Remember Pauline the pool worker? She is still working		
still working at her job maintaining pools.	at her job maintaining pools.		
2) When Pauline comes in at noon on Tuesday to fill another identical pool, fortunately she finds that there are already 1080 gallons of water in the pool. She will still fill the pool at a rate of 12 gallons per minute. This equation expresses the	3) When Pauline comes in at noon on Wednesday to fill another identical pool, the pool is empty. She gets a phone call and does not start filling the pool for 5 minutes. This equation expresses the amount of water, <i>w</i> , in the pool after <i>t</i> minutes since noon:		
amount of water, w, in the pool after t	w = 12(t - 5)		
minutes since noon:			
w = 12t + 1080	a. What does the 12 mean in the equation?b. Why is 5 subtracted from <i>t</i> in this equation?c. Write an expression for <i>w</i> that is equivalent to		
a. What does the 12 mean in the equation?	the original expression in the equation above.		

¹ Lappan, G., Phillips, E. D., Fey, J. T., Friel, S. N., Grant, Y., & Stewart, J. (2014). *Connected mathematics 3*. Boston, MA: Pearson.

b.			
	Why is 1080 added to $12t$ in the		What information does this new expression tell
	equation?	L	you about the situation?
с.	How many gallons will be in the pool	d.	How many gallons will be in the pool after 30
h	If the peel helds 5400 celleng heavy		If the need holds 5400 college, how much time
a.	much time does it take to fill the pool?	e.	doos it take to fill the pool?
9	Describe the pattern of change in the	f	Describe the pattern of change in the amount of
C.	amount of water in pool and the	1.	water in pool and the amount of minutes that
	amount of minutes that have passed		have passed
f	What does the graph for this situation	σ	What does the graph for this situation look like?
1.	look like? Can you determine what the	5.	Can you determine what the graph will look like
	graph will look like without plotting		without plotting points?
	points?	h	An engineer comes in partway through the day
ø.	An engineer comes in partway through		and figures out how many gallons are in the pool.
8.	the day and figures out how many		She does it again 15 minutes later. How much
	gallons are in the pool. She does it		will the amount of water change in that 15
	again 15 minutes later. How much will		minutes? Does it matter when during the day she
	the amount of water change in that 15		arrives?
	minutes? Does it matter when during	i.	What if the engineer measures the amount of
	the day she arrives?		water in the pool, and then measures it again $\frac{1}{2}$
			of a minute later. How much will the amount of
			water change in that 1/2 minute? Explain.
4) W	/hen Pauline comes in at noon, a pool	5) W	hen Pauline comes in at noon on Friday, a pool
ic	lentical to the one on the other days is	id	entical to the one on the other days is full. She is
fi	ill. She is supposed to empty the pool,	su	proceed to empty the pool and it empties at 17
	1		pposed to empty the pool, and it empties at 12
a	nd it empties at 12 gallons per minute.	ga	llons per minute. However, first she has to change
a T	nd it empties at 12 gallons per minute. his equation expresses the amount of	ga the	llons per minute. However, first she has to change e filter for the pool, which takes 10 minutes. This
aı T w	nd it empties at 12 gallons per minute. his equation expresses the amount of vater, w , in the pool after t minutes since	ga the eq	llons per minute. However, first she has to change e filter for the pool, which takes 10 minutes. This uation expresses the amount of water, w, in the pol after t minutes since poon:
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the amount of water change in that ½ minute? Does it matter which ½ minute she measures during the day?	h. What is the y-intercept for this graph? What could the y-intercept mean in this situation?
minute she measures during the day :	

Kathy's work on Problem #3 (Transcript), Day 22: w = 12(t-5)

RJ: So what does that point mean [where Kathy's graph crosses horizontal axis]?

Angela: Five [minutes after noon].

Kathy: No, six, so that would be 12:06 because she wasted 5 minutes.

RJ: Right where that hits right there, what is *w* [number of gallons of water in pool]? Kathy: Zero...twelve, it's twelve.

RJ: Right down here, right at this point [points to Angela's paper, where t = 5, w = 0]? Angela: That's zero.

Kathy: No, at six it would be twelve.

RJ: At six it would be twelve, but you're saying at this point where it intersects, it would be zero? Kathy: I'm confused.

RJ: If I just plotted this point right here —

Kathy: Oh wait, no, that's not correct. It would have to be up higher...