Tiering Instruction on Speed for Middle School Students

From the IDR²eAM Project: **I**nvestigating **D**ifferentiated Instruction and **R**elationships between **R**ational Number Knowl**e**dge and **A**lgebraic Reasoning in **M**iddle School

http://www.indiana.edu/~idream/

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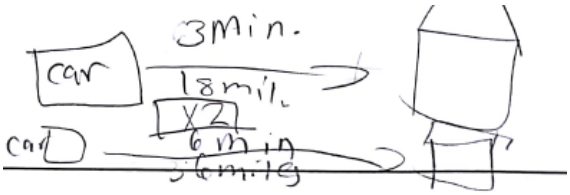
If you'd like to see a version of the app the students used, search for NewRace (one word) on geogebra.org. You can try it on a phone, although it's better on a computer or iPad. The app was developed by Janet Bowers, and the instructional design was adapted from Joanne Lobato's Math Talk project (mathtalk.sdsu.edu)

SAME SPEED TASK: The blue car travels _____ miles in _____ minutes. Make the red car travel at the same speed as the blue car, but the red car will travel a different amount of miles and a different amount of minutes. [Later we asked them to justify their results with a picture.]

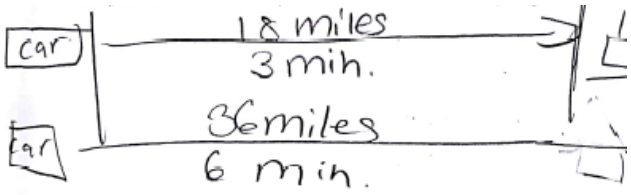
Multiplicative Concept	Orangeyness Investigation	Numbers (Blue car goes)
• MC1: Can partition lengths into equal parts but when separates them, no conceptual need to reunite parts	• Not fluidly iterating two quantities as a composed unit.	• 18 miles in 3 min • Whole number unit ratio (6 miles in 1 minute)
• MC2: Can think of lengths as parts, as a unit of units, prior to drawing	• Iterating two quantities as a composed unit.	• 15 miles in 6 min • Mixed number unit ratio with ½ (2.5 miles in 1 minute)
• MC3: Can think of length as parts containing parts, as a unit of units of units	• Iterating two quantities as a composed unit • Making unit ratios	• 15 miles in 9 min • Unit ratio hard to work with as a decimal (5/3 miles in 1 minute)

Emily, clip 1: Blue car travels 18 miles in 3 minutes

AH: You don't have to draw a car--you don't have to draw anything other than something to show 18 miles in 3 minutes. What could show it? [7-second pause] What if you want to show, say hey somebody, the car went 18 miles and it happened in 3 minutes, what would you draw to show the car's journey?
Emily [draws a car]: That's a bad car. But you have a car.
AH: Uh-huh.
Emily: And it's going [draws a line next to the car, ending at a house with "3 min" written over the line].
AH: Okay. Okay, great. I noticed, Emily, you have 3 minutes there. It's kind of like you're saying this is this distance that got covered in 3 minutes. What's the distance again?
[Emily writes "18 mil" under the line.]
AH: All right, super. So, right here you have this segment, or line, that would show that distance. Now, we also have the other car going 36 miles in 6 minutes. Do you think you could show that one?
[Emily draws another line segment below the first. She writes 6 min above the line and 36 miles below the line.]



AH: Now, would these be the same length?
Emily: No.
AH: What's the relationship between the length of this one and the length of that one?
Emily: That one's [top one] shorter than that one [bottom one].
AH: It's shorter, okay. Can you tell me anything more? How much shorter--I mean right now yours is shorter [on the paper]. Do you think it shows well that this is 18 and that's 36?
Emily: Yeah.
AH: How come?
Emily: Because it's labeling them.
AH: Do you have the idea of doubling in there though, that you were talking about?
[Emily writes "x 2" between the two drawings.]
AH: Okay, well can you show it with the lengths? Emily, can you show the doubling of the lengths? Do you get what I'm asking about?
Emily: It's two times larger.
AH: Yeah. Can you draw it that way? Maybe draw it again over here.
[Emily draws a second picture.]



Emily, clip 2: Blue car travels 18 miles in 3 minutes

AH [to Emily]: That's a really nice picture. To me, though, when I look at it, it looks like this journey and that journey are the same size [the two line segments are the same size]. Are they the same size?
Emily: No.
AH: Something's the same about them, the speed. Is there any way to show how the one journey is, as you said, twice as long, twice as big? [Emily extends the segment showing 36 miles in 6 min, but she does not make it twice as big as the segment showing the 18 miles in 3 min.]

Lisa and Sara, clip 1: Blue car travels 15 miles in 6 minutes

Sara: It's impossible.
Lisa: Ms. Hackenberg, It's impossible. We give up.
AH: I'm coming back, just a second. [She talks with another group.]
Lisa [when Ms. H returns]: It's impossible, even when we do that, 15.1 and 6.1, because it's not 15 or 6, so.
AH: Okay, so you really think it's impossible?
Lisa: Yeah.
Sara: Yeah, unless you do, like...

AH: So, two cars can't go the same exact speed but go different distances and times?
Sara: They probably could, but I can't figure it out.
Lisa: When you say we can't use 15 or 6, it's kind of hard.
AH: Right. [pause] All right, well that's good to know that it's hard.
Sara: Unless you double it, and it's going the same speed, you just doubled it.
AH: Wait, what do you mean, Sara?
Lisa: 30 and 12?
AH: Try that. What does that give you?
Sara: 30 and 12?
Lisa: Yeah, it's doubling.
AH: Double the distance and double the time. Do you think you'll go the same speed or no?
S: I don't know. [She watches as Lisa tries these numbers in the app.] I mean they're not going the same exact speed but they're going the same speed, just…
AH: Do you think it'll be the same speed, not the same exact--?
Sara [standing up]: I figured the system out!
Lisa: Oh wow, it worked! Okay.

Lisa and Sara, clip 2: Blue car travels 15 miles in 6 minutes

AH: Lisa, one thing I see in your picture is the distance from 0 to 15, right? Is this 0 right here?
Lisa: Yeah.
AH: Okay, and then onto 30. So, it looks to me like you're saying if you go 30 miles, it's like you go 15 and then another...
Lisa: 15.
AH: 15, right.
AH: So, then I wonder if that can be a beginning to show why you know that when you double, it goes the same speed. You and Sara should definitely talk.

Joanna: Blue car travels 15 miles in 9 minutes.

Mark: Wouldn't 16 and 10 work?
Joanna: I don't think it would.
Mark: Why not?
Joanna: Well, the 16 and 10, see 15 and 9 reduced would be 5 to 3, but 16…
Mark: So, it has to go [inaudible].
Joanna: It would be 5, like a ratio would be 5 to 3, and then for the red one if you did 16 and 9, 16 and 10, that would reduce to 8 to 5, and that's not 5 to 3. No, it's not.
Brianne: I want to test something.
Joanna: So, does it just have to be, sorry [to Mark]. You go.
Mark: I don't really know how to explain it but, do you get what I'm saying by 16 and 10? Because it'd be--
Joanna: Yeah, I do but it still wouldn't [work]. Sorry.
Mark: Of course. Why would I be right? I'm never right.
Joanna: I'm sorry. No, no. I just...
Mark: Joanna is always right and here to always ruin my amazing brilliant ideas.
Joanna [to Brianne and Jenni]: Do you agree with him? Because…
Mark: At least she doesn't want to hurt my feelings.
Joanna: Because they wouldn't be the same ratio to each other.
Mark: That's fine.