Inverse Reasoning with Quantitative Unknowns

From the IDR²eAM Project: **I**nvestigating **D**ifferentiated Instruction and **R**elationships between **R**ational Number Knowledge and **A**lgebraic Reasoning in **M**iddle School[1]

"Two Unknown Problems"

Fern Sunflower Heights Problem. A fern and sunflower are growing in the garden, each of unknown height. The height of the sunflower measured in inches is 3/5 the height of the fern measured in inches.

a) Draw a picture of the situation and describe what your picture represents.
b) Write an equation for this situation that relates the two heights. Explain your equation in terms of your picture.
c) Can you write another, different equation that relates the two heights? Explain this equation in terms of your picture.
d) Let's say that the fern's height is 120 cm. How tall is the sunflower?
Use this example to check all of your equations.
If an equation does not work, see if you can change it so that it does. Explain any changes that you make.

Sunflower Heights Problem. Steven and Lia each are growing a sunflower plant. The height of each of their plants is unknown. The height of Lia's plant measured in inches is 3/7 the height of Steven's plant measured in inches.

a) Draw a picture of the situation and describe what your picture represents.
b) Write an equation for this situation that relates the two heights. Explain your equation in terms of your picture.
c) Can you write another, different equation that relates the two heights? Explain this equation in terms of your picture.

Katrina's Follow-Up Interview

AH: Okay, that's fine. You have three good equations here. Do you think you can write another equation that uses a fraction? Is it possible?
K: Yeah, because there's always two type of, well, four type of different fraction and equations we can use for one equation [I think she means for one problem]. You can probably do something like… [pause] Would it work if you did something like seven thirds divided by L = S [writes "7/3 ÷ L = S"]?
AH: Well, let's find out. How do you see that in the picture? Can you generate that idea in the picture?
[K shakes head no.]
AH: No?
K: Not really because nothing is 7/3.
AH: Oh, I see. We know that Lia's height is three sevenths of Steven's, right? What fraction of Lia's height is Steven's? Do you think you can figure that out in the picture?
(00:17:14) KK [pretty promptly, not much pause]: Yeah, 2 and 1/3.
AH: How'd you figure that out?
K: [pointing at the smaller height] Because I multi—It would be like this [traces the small height] two times. Say like the height is this like three different areas [draws 4 small horizontal lines on the left edge of the paper to create 3 parts], and then you're putting three more on top of that [adds three more parts above the first three parts], so one, two, three more areas. And then there's still one more [points

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to top section of larger flower height] that's on it if you measure the heights, so that [adds one more part onto her new drawing] would be the one more area.

AH: Got it. Let's make sure I just understand. This is one [draws purple bracket around the lowest three parts], two [draws the second bracket around the next three parts], and 1/3 right here?

K: Yeah.

AH: Okay, cool. Steven's height is 2 and 1/3 of Lia's is what you're saying, right?

K: Uh-huh.

AH: Is that related at all to 7/3?

K: Yeah, because if you make it an uneven fraction, whatever it's called—

AH: Improper fraction.

K: Yeah. It would end up being 7/3.

AH: Do you think you can use that fraction? Does this [equation "7/3 ÷ L = S"] do it then? Is that the equation you would do to use 7/3 in an equation, or is there a different equation you'd write?

KK [pause]: I think that would be it because you're basically… This one—like, the thing with this type of math, you think you know it, and then you kind of like double think, double check.

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AH: People say to me sometimes this is the hard-easy math or the easy-hard math, or something like that. You're not alone in thinking that. Well, you had said to me right away you knew 2 and 1/3 and you explained how you saw that in the picture, so if you take Lia's height some number of times, what number of times do you have to take it to make Steven's height?

KK [short pause]: 2 and 1/3.

AH: Lia's height 2 and 1/3 times makes Steven's height? Would that produce an equation, that idea?

K: It would be like same thing as that [7/3 ÷ L = S], basically. [Writes 2 1/3, pauses] Wait... No, because it would be times.

AH: Which one do you think works because probably we'd have to choose between these two [7/3 ÷ L = S and 2 1/3 L = S] since this one is 7/3 or 2 and 1/3 divided by L = S, and this one is 2 and 1/3 times L = S. Which one do you think is going to work?

K: Um, I would think. [short pause] Can I solve it?