Tiered Instruction: One Strategy for Differentiation in Middle School

From the IDR²eAM Project: Investigating Differentiated Instruction and Relationships between Rational Number Knowledge and Algebraic Reasoning in Middle School

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Crate Problem: There are 4 cans of juice in a package and 8 packages in a box. A crate contains 6 boxes. How many cans of juice are in a crate, and can you draw a picture to show how you know?

Partially Filled: A worker is packing one of those crates. She has packed 2 boxes and 3 packages. How many more cans are needed to fill the crate? How will those cans be organized in the crate?

Joanna solving the Crate Problem (initial interview):

Amy: How would the crate look?
Joanna: Well, if you kind of have a sort of x-ray of it, it would be the crate. [Joanna draws .] Then inside of the crate there are 6 boxes, so, um. 6 boxes and in each box is 8 packages, so then it would just be [drawing], um... Yeah. That's 8. And then there'd be 4 cans of juice, so, how do I do this? Um. 1, 2, 3… [drawing]
Amy: I see.
Joanna: Usually it would be better if I used different colors to kind of separate things, but.

SAME SPEED TASK, end of Day 11/Day 12: The blue car travels _____ miles in _____ minutes. Make the red car travel at the same speed as the blue car, but the red car will travel a different amount of miles and a different amount of minutes. [Later we asked them to justify their results with a picture.]

Lisa and Sara's group (transcript), Day 11: Their blue car travels 15 miles in 6 minutes. They are supposed to find another distance value and time value for the red car so that it travels at the same speed.

Amy: So that's the new challenge. So, now the red car was going to try to go the same speed as the blue car but not go 15 miles in 6 minutes.
Lisa: Then it can go 14 miles in 5 minutes.
A: Try it, 14 miles in 5 minutes, see if that works. See if it makes it go the same. That would be your first trial in your table, Lisa. Sara and Ivan, do you understand what's happening here?
Sara: Yes.
A: We're going to try to make the red car go the same speed, given that the blue car is going 15 miles in 6 minutes, but the red car is going to go some other distance and time. Lisa says let's try 14 miles in five minutes and see if it goes the same speed--share with your whole table, Lisa.
Lisa: I know.
[She inputs 14 miles in 5 minutes for the red car.]
Sara: It's going faster. [pause] So it's not going to work.

***[pause of about 1.5 minutes where students try values of 15.1 miles and 6.1 min.]

Sara: It's impossible.
Lisa: Ms. Hackenberg, It's impossible. We give up.
A: I'm coming back, just a second. [She talks with another group.]
Lisa [when Ms. H returns]: It's impossible, even when we do that, 15.1 and 6.1, because it's not 15 or 6, so.
A: Okay, so you really think it's impossible?
Lisa: Yeah.
Sara: Yeah, unless you do, like...
A: So, two cars can't go the same exact speed but go different distances and times?
Sara: They probably could, but I can't figure it out.
Lisa: When you say we can't use 15 or 6, it's kind of hard.
A: Right. [pause] All right, well that's good to know that it's hard.
Sara: Unless you double it, and it's going the same speed, you just doubled it.
A: Wait, what do you mean, Sara?
Lisa: 30 and 12?
A: Try that. What does that give you?
Sara: 30 and 12?
Lisa: Yeah, it's doubling.
A: Double the distance and double the time. Do you think you'll go the same speed or no?
S: I don't know. [She watches as Lisa tries these numbers in the app.] I mean they're not going the same exact speed but they're going the same speed, just…
A: Do you think it'll be the same speed, not the same exact--?
Sara [standing up]: I figured the system out!
Lisa: Oh wow, it worked! Okay.
Ms. H [laughing]: Write that down.
Sara: I'm smart. What do I get?
Lisa: So, do we just put tie [do they write "tie" in their tables]?
Sara: Oh I did it!

Joanna's group (transcript), Day 12: Their blue car travels 15 miles in 9 minutes. They are supposed to find another distance value and time value for the red car so that it travels at the same speed.

Mark: Wouldn't 16 and 10 work?
Joanna: I don't think it would.
Mark: Why not?
Joanna: Well, the 16 and 10, see 15 and 9 reduced would be 5 to 3, but 16…
Mark: So, it has to go [inaudible].
Joanna: It would be 5, like a ratio would be 5 to 3, and then for the red one if you did 16 and 9, 16 and 10, that would reduce to 8 to 5, and that's not 5 to 3. No, it's not.
Brianne: I want to test something.
Joanna: So, does it just have to be, sorry [to Mark]. You go.
Mark: I don't really know how to explain it but, do you get what I'm saying by 16 and 10? Because it'd be--
Joanna: Yeah, I do but it still wouldn't [work]. Sorry.
Mark: Of course. Why would I be right? I'm never right.
Joanna: I'm sorry. No, no. I just...
Mark: Joanna is always right and here to always ruin my amazing brilliant ideas.
Joanna [to Brianne and Jenni]: Do you agree with him? Because…

Mark: At least she doesn't want to hurt my feelings.
Joanna: Because they wouldn't be the same ratio to each other.
Mark: That's fine.
Joanna: No, I'm sorry.
Brianne [to Mark]: You want to see if it works, 16 and 10?
Mark: Mm-hmm [yes].
Brianne: I don't think it's going to work, but just to get you to stop talking.
Mark: What's that supposed to mean?
[laughter]
Brianne: The red car is slightly behind the blue car.
Mark: Oh, man.
Joanna: Oh, man.
Brianne: Wait. [They watch again.] Yeah, it's slightly behind it. Okay, I think that 30 and 18 will work, yeah.
Jenni: That's what I was just thinking.
Joana: Do you know what I think would work? Any two numbers that will reduce to 5 and 3.
Brianne: 15?
Joanna: So, 10 and 6.
Mark: Wait. What's your idea?
Joanna: Any numbers where the miles would reduce to five and the minutes would reduce to three, because it'd be the same ratio to each other.

****[later that same day]

Joanna: My picture--I mean I don't even know if it's any good. But I kind of did a number line and I was showing how much the blue car went, which is 15 miles in 9 minutes. And I divided it up into 3 [parts], so it's 0 and then it goes 5 and 10, 15, and this goes 0, 3, to 6, to 9. And I was just showing that it goes the same distance because this goes this much and this much in the same speed. So they go the same speed; this one just stops earlier.
Rebecca: So it looks like you are saying this whole thing is that one trip of 15 miles in 9 minutes?
Joanna: Yes.
R: So what would this little piece be?
Joanna: That'd be 5 miles in 3 minutes.
R: So how does that relate to the whole trip?
Joanna: 5 miles in 3 minutes would be 1/3 of the trip.
R: A third of the trip?
Joanna: Yes.
R: So if you do that third three times you end up with the whole trip?
Joanna: Yes.
R: Does that make sense to you Jenni?
Jenni: Not really.
R: Okay. How about you, does that make sense to you Mark? Can you try explaining it to Jenni? You can use your picture or Joanna's picture. Watch his picture Jenni.
Mark: So it's 15 and 9, she pretty much just found the factors that go into each number. So you divided 15 and 9, both can be, both can be--okay, uh. She divided them both by 3. And got 3 and 5, because. Anyway, and so, and then she like multiplied them by 2 each time. 5 and 3, 10 and 6, 15 and 9.

TASK SEQUENCE, Days 9-11 (prior to same speed task)

Task 2

Make the red car go slower than the blue car when both cars travel the same distance of 15 miles

1. Go to the "Races" applet. For the blue car, enter 15 for the number of miles and 6 for the number of minutes. For the red car, enter 15 for the number of miles.
2. Make and test some predictions of the number of minutes for the red car so that the red car goes slower than the blue car when they both travel for 15 miles. **Record your results in the table in your journal.**

Answer these questions **in your journal:**

3. Did the results of any of the races surprise you? Which ones, and what surprised you?

Task 3

Make the red car go slower than the blue car when the two cars travel the same # of miles

1. Reflect on your results from Task 2. Write a rule that would tell you how to choose the number of minutes to enter for the red car so that it goes slower than the blue car when the two cars travel the same number of miles.
2. Use the "Races" applet to test your rule. Record the results **in your journal** for a few of your tests. Is your rule working? How do you know?

Answer these questions **in your journal:**

3. Is your rule working? How do you know?
4. When you run a race, how can you tell that the red car is going slower than the blue car?

Task 4

Make the red car go slower than the blue car when both cars travel for the same amount of time, 6 minutes

1. Go to the "Races" applet. For the blue car, enter 15 for the number of miles and 6 for the number of minutes. For the red car, enter 6 for the number of minutes.
2. Make and test some predictions for the number of minutes to enter for the red car so that the red car goes slower than the blue car. Record your results in the table **in your journal**.

Answer the following question **in your journal:**

3. Did the results of any of the races surprise you? Which ones, and what surprised you?

Task 5

Make the red car go slower than the blue car when the two cars travel the same # of minutes

1. Reflect on your results from Task 4. Write a rule that would tell you how to choose the number of miles to enter for the red car so it goes slower than the blue car when the two cars travel the same number of minutes. **Write this rule down in your journal!**
2. Use the "Races" applet to test your rule. Record the results for a few of your tests. Is your rule working? How do you know? **Use the table in your journal to record your results!**

Answer the following questions **in your journal:**

3. Is your rule working? How do you know?
4. When you run a race, how can you tell that the red car is going slower than the blue car?

Focus Question: The blue car goes 3 miles in 10 minutes and the red car goes ____ miles in 10 minutes. Without using the applet, give a value for the red car's number of minutes so that it would go faster. Give another value that would make it go slower. Explain how you know.